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Scalar potential from de Sitter brane in 5D and effective cosmological constant

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Abstract

We derive the scalar potential in zero mode effective action arising from a de Sitter brane embedded in five dimensions with bulk cosmological constant Λ. The scalar potential for a scalar field canonically normalized is given by the sum of exponential potentials. In the case of $\Lambda = 0$ and $\Lambda > 0$, we point out that the scalar potential has an unstable maximum at the origin and exponentially vanishes for large positive scalar field. In the case of $\Lambda < 0$, the scalar potential has an unstable maximum at the origin and a local minimum. It is shown that the positive cosmological constant in dS brane is reduced by negative potential energy of scalar at minimum and that effective cosmological constant depends on a dimensionless quantity. Furthermore, we discuss the fate of our universe including the potential energy of the scalar.

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1 Introduction

An idea of braneworld that our world may be embedded in higher dimensional world is presently developing in particle physics as well as cosmology. This is because the braneworlds have several possibilities for explaining large hierarchy between electroweak scale and Planck scale, cosmological constant problem, inflation at early universe and accelerating universe at present, *etc.* Probably, the widespread braneworld started from the Randall-Sundrum model. An model proposed by Randall and Sundrum [1] assumes that the spacetime is non-factorizable with exponential warp factor and the extra space is non compact. The remarkable feature is that a zero mode gravity can be localized on a flat 3-brane in AdS_5 background and the correction terms to the Newton potential are generated by the massive gravity with continuous modes. Moreover, the localization of gravity in the RS model have been analyzed in detail [2, 3, 4] and the extension to more higher dimensions have been performed [5, 6, 7].

It is known that the scalar potential arising from the braneworld can play an important role in cosmology and phenomenology. For instance, on searching mechanism for driving acceleration of our universe, it is expected that quintessence comes from the braneworld scenario. The possibility of quintessence in five dimensional dilatonic domain wall theory including RS model was investigated [8, 9]. The zero mode effective action with Einstein frame was studied in detail, unfortunately, it was shown that a scalar field canonically normalized cannot play the role of quintessence because of negative scalar potential. Furthermore, in the framework of string/M theory [10, 11] it was argued that the fate of our universe is determined by the properties of the scalar potential. Thus, when considering the scalar field as inflaton or quintessence in the cosmological context, it is important to study the properties of the scalar potential resulted from several models.

In present paper, we consider the model of a de Sitter brane embedded in five dimensions with bulk cosmological constant Λ [12, 13]. We extract the scalar potential from the four dimensional zero mode effective action depending on the sign of Λ and the significant feature of the potential is studied in detail. In particular we are interested in the stability of the scalar potential and the effective cosmological constant in the brane. This is because the fate of our universe is determined by the scalar potential.

The paper is organized as follows. In section 2 we explain the action of the model, moreover, the metrics depending on sign of the bulk cosmological constant are presented. In section 3 the four dimensional zero mode effective action is derived by performing integral of the extra dimension. We evaluate the scalar potential in the effective action and study the significant features of the potential depending on the sign of bulk cosmological constant. In section 4, the fate of our universe is discussed from the viewpoint of scalar cosmology. The conclusion is given in section 5. In appendix, we provide formulas of the integrals used in calculations of the effective action.

2 The model

We consider the model of a dS brane embedded in five dimensions. We start by introducing the following action [12, 13]

$$S = \int d^5x\sqrt{-G}\left(\frac{\kappa_5^{-2}}{2}\mathcal{R} - \Lambda\right) - \int d^4x\,\sqrt{-g}\,V\,, \tag{1}$$

where κ_5^2 is the five dimensional gravitational constant and Λ is the bulk cosmological constant, and V is positive brane tension. We adopt the metric as follows

$$ds^2 = G_{MN}dx^M dx^N = \Omega^{-2}(z)\left(g_{\mu\nu}(x)dx^\mu dx^\nu + \varphi^2(x)dz^2\right)\,, \tag{2}$$

where $g_{\mu\nu}$ is the four dimensional de Sitter metric with positive cosmological constant Λ_4 and z-coordinate is the fifth direction with $\mathbf{Z}_2$ symmetry. Note that $\varphi(x)$, diagonal component field for z in the metric, corresponds to the graviscalar field from the four dimensional viewpoint [9]. Depending on the sign of Λ, $\Omega(z)$ and V can be expressed as [12, 13, 14]

$$\Lambda = 0 \;\;:\;\; \Omega(z) = e^{\sqrt{\bar\Lambda}|z|}\,,\, V = 6\kappa_5^{-2}\sqrt{\bar\Lambda} \tag{3}$$

$$\Lambda > 0 \;\;:\;\; \Omega(z) = \frac{1}{L\sqrt{\bar\Lambda}}\cosh\sqrt{\bar\Lambda}(z_0+|z|)\,,\, V = \frac{6\kappa_5^{-2}}{L}\sqrt{L^2\bar\Lambda - 1} \tag{4}$$

$$\Lambda < 0 \;\;:\;\; \Omega(z) = \frac{1}{L\sqrt{\bar\Lambda}}\sinh\sqrt{\bar\Lambda}(z_0+|z|)\,,\, V = \frac{6\kappa_5^{-2}}{L}\sqrt{L^2\bar\Lambda + 1}\,, \tag{5}$$

where a dS brane is located at $z = 0$. Here we defined $3\bar\Lambda \equiv \kappa_4^2\Lambda_4$, where κ_4^2 is the four dimensional gravitational constant. Furthermore L corresponding to the bulk curvature is given by

$$L = \sqrt{\frac{6\kappa_5^{-2}}{|\Lambda|}}\,. \tag{6}$$

In addition, z_0 is given by [12, 13]

$$\Lambda > 0 \;\;:\;\; z_0 = \frac{1}{\bar\Lambda}\mathrm{arc\,cosh}\left(L\sqrt{\bar\Lambda}\right) \tag{7}$$

$$\Lambda < 0 \;\;:\;\; z_0 = \frac{1}{\bar\Lambda}\mathrm{arc\,sinh}\left(L\sqrt{\bar\Lambda}\right)\,. \tag{8}$$

In the model presented here, it was shown that the four dimensional gravity can be produced at large distance due to localization of a normalizable zero mode in gravitational fluctuation on brane [12, 13, 15].

3 Scalar potential

In this section, we calculate the four dimensional zero mode effective action for $\Lambda = 0$, $\Lambda > 0$ and $\Lambda < 0$, separately. Below, by using the appropriate conformal transformation, the effective action with Einstein frame and with a scalar field canonically normalized can be derived. Then the scalar potential is extracted from the action.

Combining (1) with (2), the action is given by

$$S = \int d^4x \sqrt{-g} L \,, \tag{9}$$

where

$$L = \int_{-\infty}^{\infty} dz \left(\frac{\kappa_5^{-2}}{2} \Omega^{-3} \varphi R + \kappa_5^{-2} \Omega^{-3} \left(4 \frac{\Omega''}{\Omega} - 10 \frac{\Omega'^2}{\Omega^2} \right) \varphi^{-1} - \Omega^{-5} \Lambda \varphi \right) - V \,. \tag{10}$$

Here the prime denotes the derivative with respect to z. By performing the integral of z in (10), we can obtain the four dimensional zero mode effective action. As indicated in [8, 9, 16], it turns out that the action corresponds to the Brans-Dicke model with zero Brans-Dicke parameter [17, 18]. The significant feature of the scalar potential in the action will be discussed below.

3.1 The case of $\Lambda = 0$

For $\Lambda = 0$, from (3), (9) and (10), the effective action is given by

$$S = \int d^4x \sqrt{-g} \left(\frac{\kappa_5^{-2}}{3\sqrt{\bar{\Lambda}}} \varphi R + 2\kappa_5^{-2} \sqrt{\bar{\Lambda}} \left(2\varphi^{-1} - 3 \right) \right) \,. \tag{11}$$

From the above action, we define the four dimensional gravitational constant as

$$\kappa_4^{-2} = \frac{2\kappa_5^{-2}}{3\sqrt{\bar{\Lambda}}} \,. \tag{12}$$

Thus the four dimensional gravitational constant is controlled by five dimensional gravitational constant and the cosmological constant in dS brane. Consequently, we have

$$S = \int d^4x \sqrt{-g} \left(\frac{\kappa_4^{-2}}{2} \varphi R + 3\kappa_4^{-2} \bar{\Lambda} \left(2\varphi^{-1} - 3 \right) \right) \,. \tag{13}$$

In order to obtain the action with Einstein frame and with a canonical kinetic term for scalar field, we need to perform the following conformal transformation

$$g_{\mu\nu} = \varphi^{-1} \bar{g}_{\mu\nu} \,, \ \varphi = e^{\sqrt{2/3}\, \kappa_4 \Phi} \,, \tag{14}$$

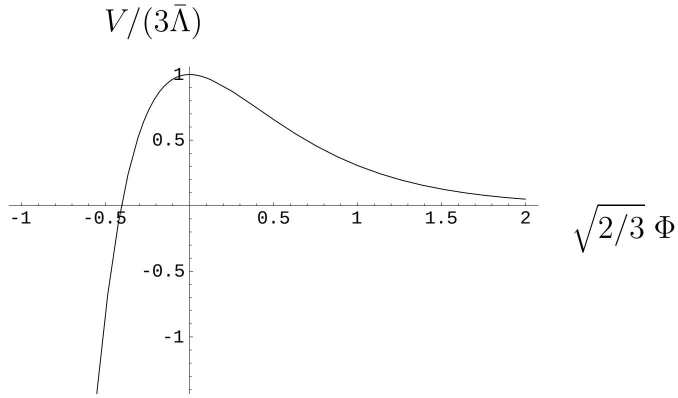

Figure 1: The scalar potential $V(\Phi)$ in the case of $\Lambda = 0$.

where a scalar field Φ is introduced. According to (14), the four dimensional zero mode effective action for a scalar field canonically normalized is given by

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{\kappa_4^{-2}}{2}\bar{\mathcal{R}} - \frac{1}{2}(\partial\Phi)^2 - V(\Phi)\right), \tag{15}$$

where the scalar potential V with $\kappa_4 = 1$ can be expressed as

$$V(\Phi) = 3\bar{\Lambda}\; e^{-2\sqrt{2/3}\,\Phi}\left(3 - 2e^{-\sqrt{2/3}\,\Phi}\right). \tag{16}$$

Thus the scalar potential can be written by the sum of exponential potentials. The graph of $V(\Phi)$ is shown in Fig.1. The potential has a maximum value $V_{\rm max} = 3\bar{\Lambda}$ at $\Phi = 0$. It is obvious that we have positive potential for $e^{\sqrt{2/3}\,\Phi} > 2/3$ and negative potential for $e^{\sqrt{2/3}\,\Phi} < 2/3$.

In the limit of $\Phi \to +\infty$, the asymptotic behavior is given by

$$V(\Phi) \approx 9\bar{\Lambda}\; e^{-2\sqrt{2/3}\,\Phi} \to +0\,. \tag{17}$$

For large positive scalar field, the potential exponentially approaches to zero. In the limit of $\Phi \to -\infty$,

$$V(\Phi) \approx -6\bar{\Lambda}\; e^{-3\sqrt{2/3}\,\Phi} \to -\infty\,. \tag{18}$$

Thus the potential goes to negative infinity for large negative scalar field.

Note that the scalar potential has a maximum at the origin, and the effective mass at the maximum is tachyonic: $m^2 = V''(0) = -12\bar{\Lambda}$. As mentioned above, there is a zero minimum far away from the origin.

3.2 The case of $\Lambda > 0$

For $\Lambda > 0$, from (4), (7) and (10), the effective action can be evaluated by using (37)$-$(39) in appendix. Therefore we have

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[\frac{1}{2}\kappa_5^{-2}L\alpha\left(\frac{\pi}{2}-\frac{\sqrt{\alpha-1}}{\alpha}-\arctan\sqrt{\alpha-1}\right)\varphi R\right.\\
&\quad +\frac{\kappa_5^{-2}}{L}\alpha^2\left(\frac{3}{4}\pi+3\frac{\sqrt{\alpha-1}}{\alpha^2}-\frac{3\sqrt{\alpha-1}}{2\alpha}-\frac{3}{2}\arctan\sqrt{\alpha-1}\right)\varphi^{-1}\\
&\quad -\Lambda L\alpha^2\left(\frac{3}{8}\pi-\frac{\sqrt{\alpha-1}}{2\alpha^2}-\frac{3\sqrt{\alpha-1}}{4\alpha}-\frac{3}{4}\arctan\sqrt{\alpha-1}\right)\varphi\\
&\quad \left.-\frac{6\kappa_5^{-2}}{L}\sqrt{\alpha-1}\right],
\end{aligned}
\tag{19}
$$

where we introduce a dimensionless quantity

$$\alpha = L^2\bar{\Lambda}\,. \tag{20}$$

As shown in the action, it is natural that the four dimensional gravitational constant is defined by

$$\kappa_4^{-2} = \kappa_5^{-2}L\alpha F\left(\alpha\right)\,, \tag{21}$$

where

$$F\left(\alpha\right) = \frac{\pi}{2}-\frac{\sqrt{\alpha-1}}{\alpha}-\arctan\sqrt{\alpha-1} \tag{22}$$

for $\alpha \geq 1$. Note that $F(\alpha)$ is monotonically decreasing function as shown in Fig.2, for example, $F(1) = \pi/2$ and $F(\infty) = 0$. Using the transformation of (14), in similar to the case of $\Lambda = 0$, after troublesome calculation the scalar potential can be extracted from the effective action with Einstein frame. Therefore we obtain

$$
\begin{aligned}
V(\Phi) &= \Lambda L e^{-\sqrt{2/3}\,\Phi}\left[\frac{3}{4}\alpha^2F(\alpha)-\frac{\sqrt{\alpha-1}}{2}+\sqrt{\alpha-1}\,e^{-\sqrt{2/3}\,\Phi}\right.\\
&\quad \left.-\left(\frac{1}{4}\alpha^2F(\alpha)+\frac{\sqrt{\alpha-1}}{2}\right)e^{-2\sqrt{2/3}\,\Phi}\right].
\end{aligned}
\tag{23}
$$

Here we set $\kappa_4 = 1$. It turns out that the scalar potential can be written by sum of exponential potentials. Next, we shall study the asymptotic behaviors of the potential for $|\Phi| \to \infty$.

In the limit of $\Phi \to +\infty$, the potential is approximated as

$$V(\Phi) \approx \frac{3}{4}\Lambda L\left(\alpha^2F(\alpha)-\frac{2}{3}\sqrt{\alpha-1}\right)e^{-\sqrt{2/3}\,\Phi}\,. \tag{24}$$

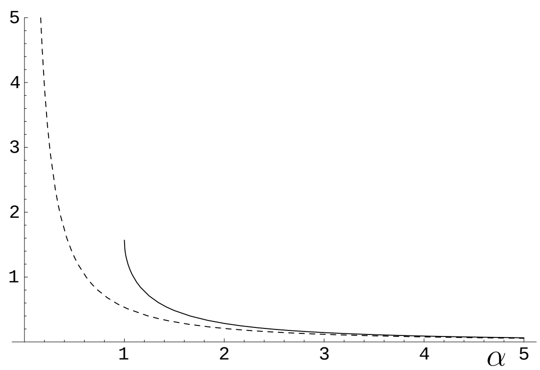

Figure 2: The graphs of $F(\alpha)$ (solid curve) and $G(\alpha)$ (broken curve).

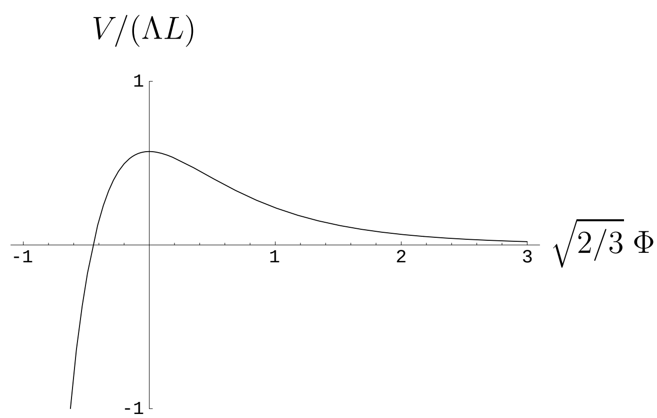

Figure 3: The scalar potential $V(\Phi)$ for $\Lambda > 0$ when $\alpha = 2$.

Note that the asymptotic behavior for $\Phi \to +\infty$ is determined by the sign of coefficient in front of exponential factor. According to the inequality $\alpha^2 F(\alpha) > \frac{2}{3}\sqrt{\alpha - 1}$ for $\alpha \geq 1$, it is clear that $V(\Phi) \to +0$ for large positive Φ. In the limit of $\Phi \to -\infty$, the scalar potential is expressed as

$$V(\Phi) \approx -\frac{\Lambda L}{4}\left(\alpha^2 F(\alpha) + 2\sqrt{\alpha - 1}\right) e^{-3\sqrt{2/3}\,\Phi} \to -\infty \,. \tag{25}$$

In order to clarify the behavior of scalar potential, the graph of $V(\Phi)$ is shown in Fig.3 when $\alpha = 2$.

Examining the scalar potential of (23), the scalar potential has a maximum at $\Phi = 0$, and we have $V_{\max} = \Lambda L \alpha^2 F(\alpha)/2$. Moreover the effective mass of scalar at the origin is tachyonic: $m^2 = V''(0) = -\frac{2}{3}\kappa_4^2 \Lambda L \left(\sqrt{\alpha - 1} + \frac{3}{2}\alpha^2 F(\alpha)\right)$. Note that the scalar potential exponentially vanishes for large positive scalar field. This implies the existence of a minimum away far from the origin.

3.3 The case of $\Lambda < 0$

For $\Lambda < 0$, from (5), (8) and (10), the effective action is evaluated by using (40)$-$(42) in appendix. Consequently, we have

$$\begin{aligned} S \;=\; \int d^4x\sqrt{-g}\,\Bigg[\, & \frac{1}{2}\kappa_5^{-2} L\alpha \left(\frac{\sqrt{1+\alpha}}{\alpha} + \log\frac{\sqrt{1+\alpha} - 1}{\sqrt{\alpha}}\right)\varphi R \\ & + \frac{\kappa_5^{-2}}{L}\alpha^2 \left(3\frac{\sqrt{1+\alpha}}{\alpha^2} + \frac{3\sqrt{1+\alpha}}{2\alpha} + \frac{3}{2}\log\frac{\sqrt{1+\alpha} - 1}{\sqrt{\alpha}}\right)\varphi^{-1} \\ & + |\Lambda| L\alpha^2 \left(\frac{\sqrt{1+\alpha}}{2\alpha^2} - \frac{3\sqrt{1+\alpha}}{4\alpha} - \frac{3}{4}\log\frac{\sqrt{1+\alpha} - 1}{\sqrt{\alpha}}\right)\varphi \\ & - \frac{6\kappa_5^{-2}}{L}\sqrt{1+\alpha}\,\Bigg] \,. \end{aligned} \tag{26}$$

In this case we define the four dimensional gravitational constant as

$$\kappa_4^{-2} = \kappa_5^{-2} L\alpha G\left(\alpha\right) \,, \tag{27}$$

where

$$G\left(\alpha\right) = \frac{\sqrt{1+\alpha}}{\alpha} + \log\frac{\sqrt{1+\alpha} - 1}{\sqrt{\alpha}} \,, \tag{28}$$

for $\alpha \geq 0$. As shown in Fig.2, $G(\alpha)$ is monotonically decreasing function. It is obvious that the action of (26) with $\bar{\Lambda} \to 0$ ($\alpha \to 0$) corresponding to a flat brane is equivalent to the effective action resulted from the RS model [8, 9, 16].

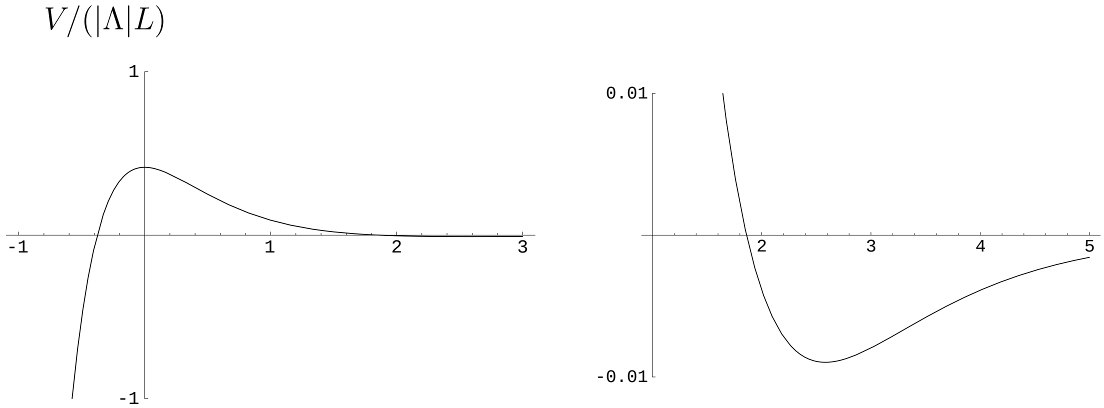

Figure 4: The scalar potential $V(\Phi)$ for $\Lambda < 0$ when $\alpha = 2$. The horizontal line is $x = \sqrt{2/3}\,\Phi$, $-1 < x < 3$ (left) and $1 < x < 5$ (right).

The transformation of (14) leads to the effective action with Einstein frame and with a scalar field canonically normalized, consequently, we can obtain the scalar potential as follows

$$\begin{aligned} V(\Phi) \;&=\; |\Lambda| L e^{-\sqrt{2/3}\,\Phi} \left[\frac{3}{4}\alpha^2 G(\alpha) - \frac{\sqrt{\alpha+1}}{2} + \sqrt{\alpha+1}\, e^{-\sqrt{2/3}\,\Phi} \right. \\ &\left. \qquad - \left(\frac{1}{4}\alpha^2 G(\alpha) + \frac{\sqrt{\alpha+1}}{2} \right) e^{-2\sqrt{2/3}\,\Phi} \right]. \end{aligned} \tag{29}$$

Here we set $\kappa_4 = 1$. Comparing (29) with (23), (29) is consistent with the equation with replacements of $F \to G$ and $\sqrt{\alpha - 1} \to \sqrt{\alpha + 1}$ in (23).

In the limit of $\Phi \to +\infty$, the potential is given by

$$V(\Phi) \approx \frac{3}{4}|\Lambda| L \left(\alpha^2 G(\alpha) - \frac{2}{3}\sqrt{1+\alpha} \right) e^{-\sqrt{2/3}\,\Phi}\,. \tag{30}$$

From inequality $\alpha^2 G(\alpha) < \frac{2}{3}\sqrt{1+\alpha}$ for $\alpha \geq 0$, it turns out that $V(\Phi) \to -0$ for $\Phi \to +\infty$. Taking the limit of $\Phi \to -\infty$, we have

$$V(\Phi) \approx -\frac{|\Lambda| L}{4} \left(\alpha^2 G(\alpha) + 2\sqrt{1+\alpha} \right) e^{-3\sqrt{2/3}\,\Phi} \to -\infty\,. \tag{31}$$

The graph of the scalar potential is shown in Fig.4 when $\alpha = 2$.

Examining (29), it turns out that the scalar potential has two extremum at $\Phi = 0, \Phi_0$. At $\Phi = 0$, there is a maximum with tachyonic mass:

$$m^2 = -\frac{2}{3}\kappa_4^2 |\Lambda| L \left(\sqrt{\alpha+1} + \frac{3}{2}\alpha^2 G(\alpha) \right). \tag{32}$$

Furthermore, there exists a local minimum at Φ_0:

$$\Phi_0 = \sqrt{\frac{3}{2}} \log\left(3\frac{2\sqrt{\alpha+1}+\alpha^2 G(\alpha)}{2\sqrt{\alpha+1}-3\alpha^2 G(\alpha)}\right) . \tag{33}$$

Note that the value of the minimum at Φ_0 is always negative and the effective mass is given by

$$V''(\Phi_0) = \kappa_4^2|\Lambda| L e^{-\sqrt{6}\,\Phi_0}\, \frac{2\sqrt{\alpha+1}+3\alpha^2 G(\alpha)}{2\sqrt{\alpha+1}-3\alpha^2 G(\alpha)} \left(2\sqrt{\alpha+1}+\alpha^2 G(\alpha)\right) > 0 \, . \tag{34}$$

Accordingly this implies a local minimum at Φ_0. The positive cosmological constant in dS brane is influenced by contribution of potential energy of scalar field at minimum, namely, the effective cosmological constant is given by $\Lambda_{\rm eff} = \Lambda_4 + V(\Phi_0)$. Therefore we obtain

$$\Lambda_{\rm eff} = \frac{|\Lambda| L}{2}\left(\alpha^2 G(\alpha) - \frac{4\sqrt{\alpha+1}+3\alpha^2 G(\alpha)}{27}\left(\frac{2\sqrt{\alpha+1}-3\alpha^2 G(\alpha)}{4\sqrt{\alpha+1}+\alpha^2 G(\alpha)}\right)^2\right) . \tag{35}$$

The second term corresponds to the potential energy of scalar field at minimum. Thus the positive cosmological constant in dS brane is reduced by dynamics of scalar field. Estimating (35), $\Lambda_{\rm eff}$ is monotonically increasing function for α. Note that $\Lambda_{\rm eff} < 0$ for $0 \le \alpha \lesssim 0.04$ and $\Lambda_{\rm eff} > 0$ for $0.04 < \alpha$. Thus it is considered that the value of the effective cosmological constant in dS brane can be tuned via scalar dynamics on the dS brane.

4 Cosmological implications

We would like to discuss the fate of the universe in the framework of this model, assuming that our universe is dominated by a scalar field Φ. It is natural to consider a scalar Φ as the component of dark energy driving acceleration of the universe at present.

Here we review the important role of the scalar potential in cosmological model [20]. Particularly, two types are presented: I) The scalar potential V slowly decreases from positive to zero as a scalar rolls to infinity (slow-roll). Assuming that dark energy corresponds to the energy of a slowly rolling scalar with equation of state ($p_D \sim -\rho_D$), the universe goes from dS regime (the present stage of acceleration) to Minkowski regime. In the case of exponential potential, Quintessence requires $V(\phi) \sim e^{-\lambda\phi}$, where $\lambda < \sqrt{2}$. II) The scalar potential V has a negative minimum or it falls to the negative infinity (free-fall). In this case the fate of the universe reaches the stage of collapse [21]. Below we apply the present model to the above two types.

The total potential energy in the dS brane is expressed as $V_{\rm tot} = \Lambda_4 + V(\Phi)$, where Λ_4 is the cosmological constant in dS brane, assuming that Λ_4 is a very small positive value. In this paper the origin of fixing the magnitude of Λ_4 isn't discussed. Namely the plots of $V(\Phi)$ in Fig 1,3,4 are lifted by Λ_4. Investigating total potential energy $V_{\rm tot}$ in the present model, we obtain the results as follows.

i) For $\Lambda = 0$ the potential has both properties of slow-roll and free-fall as shown in Fig.1. The roll from the top to $\Phi = +\infty$ slowly decreases, however, acceleration cannot arise because of $V_{\rm tot} \simeq e^{-\lambda\Phi}$ with $\lambda = 2\sqrt{2/3} > \sqrt{2}$ from (17). The roll to $\Phi < 0$ is free to fall to $V_{\rm tot} = -\infty$, namely, it implies collapse of the universe.

ii) For $\Lambda > 0$, as shown in Fig.3, the potential has both properties of slow-roll and free-fall. The roll from the top to $\Phi = +\infty$ can generate acceleration because of $V_{\rm tot} \simeq e^{-\lambda\Phi}$ with $\lambda = \sqrt{2/3} < \sqrt{2}$ from (24). While the roll from the top to $V_{\rm tot} = -\infty$ leads to collapse of the universe.

iii) For $\Lambda < 0$, from Fig.4, the potential has the property of free-fall and a minimum obtained in (35). The roll of free-fall implies collapse of the universe. The rolling scalar from the top to a minimum or from $\Phi = +\infty$ to a minimum would reach a minimum $\Lambda_{\rm eff}$ of (35), however, in this case the fate of the universe depends on the sign of $\Lambda_{\rm eff}$. The case of negative $\Lambda_{\rm eff}$ ($0 < \alpha < 0.04$) implies a negative minimum, namely, the universe collapses. Although the case of positive $\Lambda_{\rm eff}$ ($\alpha > 0.04$) leads to dS regime during present stage of acceleration, the scalar is generically tunneling into negative infinity and the universe eventually goes to collapse.

5 Conclusion

In this paper we have studied the scalar potential by evaluating the four dimensional zero mode effective action resulting from the model of a dS brane embedded in five dimensions with bulk cosmological constant Λ. The scalar potential is explored in the case of $\Lambda = 0$, $\Lambda > 0$ and $\Lambda < 0$, separately. We pointed out that the potential for a scalar field canonically normalized is given by the sum of exponential potentials and discussed the fate of our universe from the viewpoint of scalar dominated cosmology.

For $\Lambda = 0$ and $\Lambda > 0$, the scalar potential has an unstable maximum at the origin, and both properties of free-fall and slow-roll are shown. In the case of $\Lambda > 0$, the scalar potential can be peculiarly suitable for Quintessence potential.

For $\Lambda < 0$, the potential has an unstable maximum at the origin and a local minimum at Φ_0. If we are living in the minimum, the positive cosmological constant in dS brane can be reduced by negative potential energy of scalar field at minimum. Consequently it turned out that the effective cosmological constant in the brane depending on a dimensionless quantity α in (20) is tuned via the dynamics of scalar field. In this case the fate of the universe eventually would collapse via tunneling effect even if we

are living in the local minimum (false vacuum).

Advanced astronomical observations [19] indicate that our universe is accelerating today and that the cosmological constant has sufficiently small positive value. In toy model presented here, the value of the cosmological constant can be controlled by a dimensionless quantity α so as to be consistent with the observable value. However this is not to say that we solve the cosmological constant problem. This is because the mechanism for determining the value of α is unknown. Thus, in the development of cosmology [22], we think that it is important to investigate scalar potential resulted from braneworld.

Finally we give some comments with regard to cosmological context. We have to study not only the form of the scalar potential, but also the scalar evolution in the model. By analyzing the scalar evolution, we expect that the fate of our universe can be discussed in detail. We will describe it elsewhere.

Appendix: Evaluation of integrals including hyperbolic functions

In Appendix, we provide formulas of integral calculations performed in section 3. Below all integral constants are omitted.

For $m, n \in \mathbf{Z}$, we define $I[m, n]$ by

$$I[m,n] = \int \sinh^m x \ \cosh^n x \ dx \,. \tag{36}$$

In section 3, the zero mode effective action can be obtained by using formulas as follows

$$\begin{aligned}
I[0,-3] &= \frac{\sinh x}{2\cosh^2 x} + \frac{1}{2}\mathrm{arc}\tan\left(\sinh x\right)\ , && (37)\\
I[0,-5] &= \frac{\sinh x}{4\cosh^4 x} + \frac{3\sinh x}{8\cosh^2 x} + \frac{3}{8}\mathrm{arc}\tan\left(\sinh x\right)\ , && (38)\\
I[2,-5] &= -\frac{\sinh x}{4\cosh^4 x} + \frac{\sinh x}{8\cosh^2 x} + \frac{1}{8}\mathrm{arc}\tan\left(\sinh x\right)\ , && (39)\\
I[-3,0] &= -\frac{\cosh x}{2\sinh^2 x} - \frac{1}{2}\log\left|\tanh\frac{x}{2}\right|\ , && (40)\\
I[-5,0] &= -\frac{\cosh x}{4\sinh^4 x} + \frac{3\cosh x}{8\sinh^2 x} + \frac{3}{8}\log\left|\tanh\frac{x}{2}\right|\ , && (41)\\
I[-5,2] &= -\frac{\cosh x}{4\sinh^4 x} - \frac{\cosh x}{8\sinh^2 x} - \frac{1}{8}\log\left|\tanh\frac{x}{2}\right|\ . && (42)
\end{aligned}$$

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